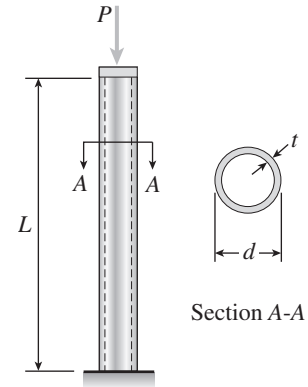


Problem 11.9-9 Determine the allowable axial load P_{allow} for a steel pipe column that is fixed at the base and free at the top (see figure) for each of the following lengths: $L = 6$ ft, 9 ft, 12 ft, and 15 ft. The column has outside diameter $d = 6.625$ in. and wall thickness $t = 0.280$ in. (Assume $E = 29,000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-9 through 11.9-12

Solution 11.9-9 Steel pipe column

Fixed-free column ($K = 2$).

Use AISC formulas.

$$d_2 = 6.625 \text{ in.} \quad t = 0.280 \text{ in.} \quad d_1 = 6.065 \text{ in.}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 5.5814 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 28.142 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 2.2455 \quad \left(\frac{KL}{r}\right)_{\text{max}} = 200$$

$$E = 29,000 \text{ ksi} \quad \sigma_Y = 36 \text{ ksi}$$

$$\text{Eq. (11-76):} \quad \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$$L_c = 126.1 \frac{r}{K} = 141.6 \text{ in.} = 11.8 \text{ ft}$$

L	6 ft	9 ft	12 ft	15 ft
KL/r	64.13	96.19	128.3	160.3
n_1 (Eq. 11-79)	1.841	1.897	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.4730	0.3737	–	–
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	–	–	0.2519	0.1614
σ_{allow} (ksi)	17.03	13.45	9.078	5.810
$P_{\text{allow}} = A \sigma_{\text{allow}}$	95.0 k	75.1 k	50.7 k	32.4 k

Problem 11.9-10 Determine the allowable axial load P_{allow} for a steel pipe column that is fixed at the base and free at the top (see figure) for each of the following lengths: $L = 2.6$ m, 2.8 m, 3.0 m, and 3.2 m. The column has outside diameter $d = 140$ mm and wall thickness $t = 7$ mm. (Assume $E = 200$ GPa and $\sigma_Y = 250$ MPa.)

Solution 11.9-10 Steel pipe column

Fixed-free column ($K = 2$).

Use AISC formulas.

$$d_2 = 140 \text{ mm} \quad t = 7.0 \text{ mm} \quad d_1 = 126 \text{ mm}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2924.8 \text{ mm}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 6.4851 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 47.09 \text{ mm} \quad \left(\frac{KL}{r}\right)_{\text{max}} = 200$$

$$E = 200 \text{ GPa} \quad \sigma_Y = 250 \text{ MPa}$$

$$\text{Eq. (11-76):} \quad \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$$

$$L_c = 125.7 \frac{r}{K} = 2959 \text{ mm} = 2.959 \text{ m}$$

L	2.6 m	2.8 m	3.0 m	3.2 m
KL/r	110.4	118.9	127.4	135.9
n_1 (Eq. 11-79)	1.911	1.916	–	–
n_2 (Eq. 11-80)	–	–	1.917	1.917
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.3212	0.2882	–	–
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	–	–	0.2537	0.2230
σ_{allow} (MPa)	80.29	72.06	63.43	55.75
$P_{\text{allow}} = A \sigma_{\text{allow}}$	235 kN	211 kN	186 kN	163 kN

Problem 11.9-11 Determine the maximum permissible length L_{\max} for a steel pipe column that is fixed at the base and free at the top and must support an axial load $P = 40$ k (see figure). The column has outside diameter $d = 4.0$ in., wall thickness $t = 0.226$ in., $E = 29,000$ ksi, and $\sigma_Y = 42$ ksi.

Solution 11.9-11 Steel pipe column

Fixed-free column ($K = 2$). $P = 40$ k
Use AISC formulas.

$$d_2 = 4.0 \text{ in.} \quad t = 0.226 \text{ in.} \quad d_1 = 3.548 \text{ in.}$$

$$A = \frac{\pi}{4} (d_2^2 - d_1^2) = 2.6795 \text{ in.}^2$$

$$I = \frac{\pi}{64} (d_2^4 - d_1^4) = 4.7877 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.3367 \quad \left(\frac{KL}{r}\right)_{\max} = 200$$

$$E = 29,000 \text{ ksi} \quad \sigma_Y = 42 \text{ ksi}$$

$$\text{Eq. (11-76):} \quad \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$$

$$L_c = 116.7 \frac{r}{K} = 78.03 \text{ in.} = 6.502 \text{ ft}$$

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{\text{allow}} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

If $L > L_c$, use Eqs. (11-80) and (11-82).

$L(\text{ft})$	5.20	5.25	5.23
KL/r	93.86	94.26	93.90
n_1 (Eq. 11-79)	1.903	1.904	1.903
n_2 (Eq. 11-80)	–	–	–
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.3575	0.3541	0.3555
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	–	–	–
σ_{allow} (ksi)	15.02	14.87	14.93
$P_{\text{allow}} = A \sigma_{\text{allow}}$	40.2 k	39.8 k	40.0 k

For $P = 40$ k, $L_{\max} = 5.23$ ft ←

Problem 11.9-12 Determine the maximum permissible length L_{\max} for a steel pipe column that is fixed at the base and free at the top and must support an axial load $P = 500$ kN (see figure). The column has outside diameter $d = 200$ mm, wall thickness $t = 10$ mm, $E = 200$ GPa, and $\sigma_Y = 250$ MPa.

Solution 11.9-12 Steel pipe column

Fixed-free column ($K = 2$). $P = 500$ kN
Use AISC formulas.

$$d_2 = 200 \text{ mm} \quad t = 10 \text{ mm} \quad d_1 = 180 \text{ mm}$$

$$A = \frac{\pi}{4} (d_2^2 - d_1^2) = 5,969.0 \text{ mm}^2$$

$$I = \frac{\pi}{64} (d_2^4 - d_1^4) = 27.010 \times 10^6 \text{ mm}^4$$

$$r = \sqrt{\frac{I}{A}} = 67.27 \text{ mm} \quad \left(\frac{KL}{r}\right)_{\max} = 200$$

$$E = 200 \text{ GPa} \quad \sigma_Y = 250 \text{ MPa}$$

$$\text{Eq. (11-76):} \quad \left(\frac{KL}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 125.7$$

$$L_c = 125.7 \frac{r}{K} = 4.226 \text{ m}$$

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{\text{allow}} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

If $L > L_c$, use Eqs. (11-80) and (11-82).

$L(\text{m})$	3.55	3.60	3.59
KL/r	105.5	107.0	106.7
n_1 (Eq. 11-79)	1.908	1.909	1.909
n_2 (Eq. 11-80)	–	–	–
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.3393	0.3338	0.3349
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	–	–	–
σ_{allow} (MPa)	84.83	83.46	83.74
$P_{\text{allow}} = A \sigma_{\text{allow}}$	506 kN	498 kN	500 kN

For $P = 500$ kN, $L = 3.59$ m ←

Problem 11.9-13 A steel pipe column with *pinned ends* supports an axial load $P = 21$ k. The pipe has outside and inside diameters of 3.5 in. and 2.9 in., respectively. What is the maximum permissible length L_{\max} of the column if $E = 29,000$ ksi and $\sigma_Y = 36$ ksi?

Solution 11.9-13 Steel pipe column

Pinned ends ($K = 1$). $P = 21$ k

Use AISC formulas.

$$d_2 = 3.5 \text{ in.} \quad t = 0.3 \text{ in.} \quad d_1 = 2.9 \text{ in.}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 3.0159 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 3.8943 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.1363 \text{ in.} \quad \left(\frac{L}{r}\right)_{\max} = 200$$

$$E = 29,000 \text{ ksi} \quad \sigma_Y = 36 \text{ ksi}$$

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$$L_c = 126.1 r = 143.3 \text{ in.} = 11.9 \text{ ft}$$

Select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{\text{allow}} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

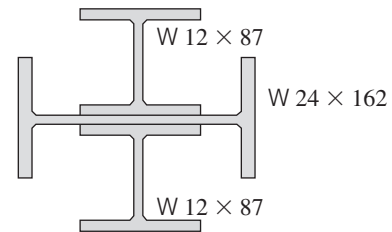
If $L > L_c$, use Eqs. (11-80) and (11-82).

L (ft)	13.8	13.9	14.0
L/r	145.7	146.8	147.8
n_1 (Eq. 11-79)	—	—	—
n_2 (Eq. 11-80)	1.917	1.917	1.917
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	—	—	—
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	0.1953	0.1925	0.1898
σ_{allow} (ksi)	7.031	6.931	6.832
$P_{\text{allow}} = A \sigma_{\text{allow}}$	21.2 k	20.9 k	20.6 k

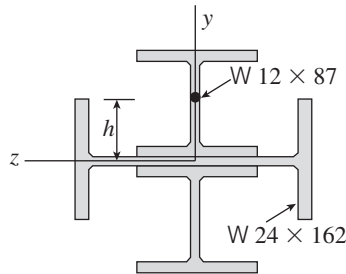
For $P = 21$ k, $L = 13.9$ ft ←

Problem 11.9-14 The steel columns used in a college recreation center are 55 ft long and are formed by welding three wide-flange sections (see figure). The columns are pin-supported at the ends and may buckle in any direction.

Calculate the allowable load P_{allow} for one column, assuming $E = 29,000$ ksi and $\sigma_Y = 36$ ksi.



Solution 11.9-14 Pinned-end column ($K = 1$)



$$L = 55 \text{ ft} = 660 \text{ in.}$$

$$E = 29,000 \text{ ksi}$$

$$\sigma_Y = 36 \text{ ksi}$$

$$W 12 \times 87$$

$$A = 25.6 \text{ in.}^2 \quad d = 12.53 \text{ in.}$$

$$I_1 = 740 \text{ in.}^4 \quad I_2 = 241 \text{ in.}^4$$

$$W 24 \times 162$$

$$A = 47.7 \text{ in.}^2 \quad t_w = 0.705 \text{ in.}$$

$$I_1 = 5170 \text{ in.}^4 \quad I_2 = 443 \text{ in.}^4$$

FOR THE ENTIRE CROSS SECTION

$$A = 2(25.6) + 47.7 = 98.9 \text{ in.}^2$$

$$I_y = 2(241) + 5170 = 5652 \text{ in.}^4$$

$$h = d/2 + t_w/2 = 6.6175 \text{ in.}$$

$$I_z = 443 + 2[740 + (25.6)(6.6175)^2] = 4165 \text{ in.}^4$$

$$\min. r = \sqrt{\frac{I_z}{A}} = \sqrt{\frac{4165}{98.9}} = 6.489 \text{ in.}$$

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

(Continued)

$$\frac{L}{r} = \frac{660 \text{ in.}}{6.489 \text{ in.}} = 101.7 \quad \frac{L}{r} < \left(\frac{L}{r}\right)_c$$

∴ Use Eqs. (11-79) and (11-81).

$$\text{From Eq. (11-79): } n_1 = 1.904$$

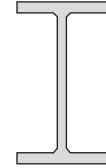
$$\text{From Eq. (11-81): } \sigma_{\text{allow}}/\sigma_Y = 0.3544$$

$$\sigma_{\text{allow}} = 0.3544 \sigma_Y = 12.76 \text{ ksi}$$

$$P_{\text{allow}} = \sigma_{\text{allow}} A = (12.76 \text{ ksi})(98.9 \text{ in.}^2)$$

$$= 1260 \text{ k} \quad \leftarrow$$

Problem 11.9-15 A W 8 × 28 steel wide-flange column with pinned ends carries an axial load P . What is the maximum permissible length L_{max} of the column if (a) $P = 50 \text{ k}$, and (b) $P = 100 \text{ k}$? (Assume $E = 29,000 \text{ ksi}$ and $\sigma_Y = 36 \text{ ksi}$.)



Probs. 11.9-15 and 11.9-16

Solution 11.9-15 Steel wide-flange column

Pinned ends ($K = 1$).

Buckling about axis 2-2 (see Table E-1).

Use AISC formulas.

$$\text{W } 8 \times 28 \quad A = 8.25 \text{ in.}^2 \quad r_2 = 1.62 \text{ in.}$$

$$E = 29,000 \text{ ksi} \quad \sigma_Y = 36 \text{ ksi} \quad \left(\frac{L}{r}\right)_{\text{max}} = 200$$

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1$$

$$L_c = 126.1 r = 204.3 \text{ in.} = 17.0 \text{ ft}$$

For each load P , select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{\text{allow}} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

If $L > L_c$, use Eqs. (11-80) and (11-82).

(a) $P = 50 \text{ k}$

L (ft)	21.0	21.5	21.2
L/r	155.6	159.3	157.0
n_1 (Eq. 11-79)	—	—	—
n_2 (Eq. 11-80)	1.917	1.917	1.917
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	—	—	—
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	0.1714	0.1635	0.1682
σ_{allow} (ksi)	6.171	5.888	6.056
$P_{\text{allow}} = A \sigma_{\text{allow}}$	50.9 k	48.6 k	50.0 k

$$\text{For } P = 50 \text{ k, } L_{\text{max}} = 21.2 \text{ ft} \quad \leftarrow$$

(b) $P = 100 \text{ k}$

L (ft)	14.3	14.4	14.5
L/r	105.9	106.7	107.4
n_1 (Eq. 11-79)	1.908	1.908	1.909
n_2 (Eq. 11-80)	—	—	—
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.3393	0.3366	0.3338
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	—	—	—
σ_{allow} (ksi)	12.21	12.12	12.02
$P_{\text{allow}} = A \sigma_{\text{allow}}$	100.8 k	100.0 k	99.2 k

$$\text{For } P = 100 \text{ k, } L_{\text{max}} = 14.4 \text{ ft} \quad \leftarrow$$

Problem 11.9-16 A $W 10 \times 45$ steel wide-flange column with pinned ends carries an axial load P . What is the maximum permissible length L_{\max} of the column if (a) $P = 125$ k, and (b) $P = 200$ k? (Assume $E = 29,000$ ksi and $\sigma_Y = 42$ ksi.)

Solution 11.9-16 Steel wide-flange column

Pinned ends ($K = 1$).

Buckling about axis 2-2 (see Table E-1).

Use AISC formulas.

$W 10 \times 45$ $A = 13.3 \text{ in.}^2$ $r_2 = 2.01 \text{ in.}$

$$E = 29,000 \text{ ksi} \quad \sigma_Y = 42 \text{ ksi} \quad \left(\frac{L}{r}\right)_{\max} = 200$$

$$\text{Eq. (11-76): } \left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7$$

$$L_c = 116.7 r = 235 \text{ in.} = 19.6 \text{ ft}$$

For each load P , select trial values of the length L and calculate the corresponding values of P_{allow} (see table). Interpolate between the trial values to obtain the value of L that produces $P_{\text{allow}} = P$.

Note: If $L < L_c$, use Eqs. (11-79) and (11-81).

If $L > L_c$, use Eqs. (11-80) and (11-82).

(a) $P = 125$ k

L (ft)	21.0	21.1	21.2
L/r	125.4	126.0	126.6
n_1 (Eq. 11-79)	–	–	–
n_2 (Eq. 11-80)	1.917	1.917	1.917
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	–	–	–
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	0.2262	0.2241	0.2220
σ_{allow} (ksi)	9.500	9.411	9.322
$P_{\text{allow}} = A \sigma_{\text{allow}}$	126.4 k	125.2 k	124.0 k

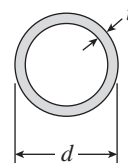
For $P = 125$ k, $L_{\max} = 21.1 \text{ ft}$ ←

(b) $P = 200$ k

L (ft)	15.5	15.6	15.7
L/r	92.54	93.13	93.73
n_1 (Eq. 11-79)	1.902	1.902	1.903
n_2 (Eq. 11-80)	–	–	–
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.3607	0.3584	0.3561
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	–	–	–
σ_{allow} (ksi)	15.15	15.05	14.96
$P_{\text{allow}} = A \sigma_{\text{allow}}$	201.5 k	200.2 k	198.9 k

For $P = 200$ k, $L_{\max} = 15.6 \text{ ft}$ ←

Problem 11.9-17 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 20$ ft that is pinned at both ends and must support an axial load $P = 25$ k. Assume that the wall thickness t is equal to $d/20$. (Use $E = 29,000$ ksi and $\sigma_Y = 36$ ksi.)



Probs. 11.9-17 through 11.9-20

Solution 11.9-17 Pipe column

Pinned ends ($K = 1$).

$L = 20 \text{ ft} = 240 \text{ in.}$ $P = 25 \text{ k}$

$d =$ outside diameter $t = d/20$

$E = 29,000 \text{ ksi}$ $\sigma_Y = 36 \text{ ksi}$

$$A = \frac{\pi}{4} [d^2 - (d - 2t)^2] = 0.14923 d^2$$

$$I = \frac{\pi}{64} [d^4 - (d - 2t)^4] = 0.016881 d^4$$

$$r = \sqrt{\frac{I}{A}} = 0.33634 d$$

(Continued)

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 126.1 \quad L_c = (126.1)r$$

Select various values of diameter d until we obtain $P_{\text{allow}} = P$.

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).

If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

d (in.)	4.80	4.90	5.00
A (in. ²)	3.438	3.583	3.731
I (in. ⁴)	8.961	9.732	10.551
r (in.)	1.614	1.648	1.682
L_c (in.)	204	208	212
L/r	148.7	145.6	142.7
n_2 (Eq. 11-80)	23/12	23/12	23/12
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-82)	0.1876	0.1957	0.2037
σ_{allow} (ksi)	6.754	7.044	7.333
$P_{\text{allow}} = A \sigma_{\text{allow}}$	23.2 k	25.2 k	27.4 k

For $P = 25$ k, $d = 4.89$ in. ←

Problem 11.9-18 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 3.5$ m that is pinned at both ends and must support an axial load $P = 130$ kN. Assume that the wall thickness t is equal to $d/20$. (Use $E = 200$ GPa and $\sigma_Y = 275$ MPa).

Solution 11.9-18 Pipe column

Pinned ends ($K = 1$).

$$L = 3.5 \text{ m} \quad P = 130 \text{ kN}$$

$$d = \text{outside diameter} \quad t = d/20$$

$$E = 200 \text{ GPa} \quad \sigma_Y = 275 \text{ MPa}$$

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2] = 0.14923 d^2$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] = 0.016881 d^4$$

$$r = \sqrt{\frac{I}{A}} = 0.33634 d$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 119.8 \quad L_c = (119.8)r$$

Select various values of diameter d until we obtain

$$P_{\text{allow}} = P.$$

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).

If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

d (mm)	98	99	100
A (mm ²)	1433	1463	1492
I (mm ⁴)	1557×10^3	1622×10^3	1688×10^3
r (mm)	32.96	33.30	33.64
L_c (mm)	3950	3989	4030
L/r	106.2	105.1	104.0
n_1 (Eq. 11-79)	1.912	1.911	1.910
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.3175	0.3219	0.3263
σ_{allow} (MPa)	87.32	88.53	89.73
$P_{\text{allow}} = A \sigma_{\text{allow}}$	125.1 kN	129.5 kN	133.9 kN

For $P = 130$ kN, $d = 99$ mm ←

Problem 11.9-19 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 11.5$ ft that is pinned at both ends and must support an axial load $P = 80$ k. Assume that the wall thickness t is 0.30 in. (Use $E = 29,000$ ksi and $\sigma_Y = 42$ ksi.)

Solution 11.9-19 Pipe column

Pinned ends ($K = 1$).

$$L = 11.5 \text{ ft} = 138 \text{ in.} \quad P = 80 \text{ k}$$

$$d = \text{outside diameter} \quad t = 0.30 \text{ in.}$$

$$E = 29,000 \text{ ksi} \quad \sigma_Y = 42 \text{ ksi}$$

$$A = \frac{\pi}{4}[d^2 - (d - 2t)^2]$$

$$I = \frac{\pi}{64}[d^4 - (d - 2t)^4] \quad r = \sqrt{\frac{I}{A}}$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 116.7 \quad L_c = (116.7)r$$

Select various values of diameter d until we obtain

$$P_{\text{allow}} = P.$$

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).
 If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

d (in.)	5.20	5.25	5.30
A (in. ²)	4.618	4.665	4.712
I (in. ⁴)	13.91	14.34	14.78
r (in.)	1.736	1.753	1.771
L_c (in.)	203	205	207
L/r	79.49	78.72	77.92
n_1 (Eq. 11-79)	1.883	1.881	1.880
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.4079	0.4107	0.4133
σ_{allow} (ksi)	17.13	17.25	17.36
$P_{\text{allow}} = A \sigma_{\text{allow}}$	79.1 k	80.5 k	81.8 k

For $P = 80$ k, $d = 5.23$ in. ←

Problem 11.9-20 Find the required outside diameter d for a steel pipe column (see figure) of length $L = 3.0$ m that is pinned at both ends and must support an axial load $P = 800$ kN. Assume that the wall thickness t is 9 mm. (Use $E = 200$ GPa and $\sigma_Y = 300$ MPa.)

Solution 11.9-20 Pipe column

Pinned ends ($K = 1$).

$L = 3.0$ m $P = 800$ kN

$d =$ outside diameter $t = 9.0$ mm

$E = 200$ GPa $\sigma_Y = 300$ MPa

$$A = \frac{\pi}{4} [d^2 - (d - 2t)^2]$$

$$I = \frac{\pi}{64} [d^4 - (d - 2t)^4] \quad r = \sqrt{\frac{I}{A}}$$

$$\left(\frac{L}{r}\right)_c = \sqrt{\frac{2\pi^2 E}{\sigma_Y}} = 114.7 \quad L_c = (114.7)r$$

Select various values of diameter d until we obtain $P_{\text{allow}} = P$.

If $L \leq L_c$, Use Eqs. (11-79) and (11-81).

If $L \geq L_c$, Use Eqs. (11-80) and (11-82).

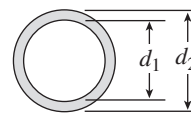
d (mm)	193	194	195
A (mm ²)	5202	5231	5259
I (mm ⁴)	20.08×10^6	22.43×10^6	22.80×10^6
r (mm)	65.13	65.48	65.84
L_c (mm)	7470	7510	7550
L/r	46.06	45.82	45.57
n_1 (Eq. 11-79)	1.809	1.809	1.808
$\sigma_{\text{allow}}/\sigma_Y$ (Eq. 11-81)	0.5082	0.5087	0.5094
σ_{allow} (MPa)	152.5	152.6	152.8
$P_{\text{allow}} = A \sigma_{\text{allow}}$	793.1 kN	798.3 kN	803.8 kN

For $P = 800$ kN, $d = 194$ mm ←

Aluminum Columns

Problem 11.9-21 An aluminum pipe column (alloy 2014-T6) with pinned ends has outside diameter $d_2 = 5.60$ in. and inside diameter $d_1 = 4.80$ in. (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 6$ ft, 8 ft, 10 ft, and 12 ft.



Probs. 11.9-21 through 11.9-24

Solution 11.9-21 Aluminum pipe column

Alloy 2014-T6

Pinned ends ($K = 1$).

$d_2 = 5.60$ in.

$d_1 = 4.80$ in.

$$A = \frac{\pi}{4} (d_2^2 - d_1^2) = 6.535 \text{ in.}^2$$

$$I = \frac{\pi}{64} (d_2^4 - d_1^4) = 22.22 \text{ in.}^4$$

(Continued)

$$r = \sqrt{\frac{I}{A}} = 1.844 \text{ in.}$$

Use Eqs. (11-84 *a* and *b*):

$$\sigma_{\text{allow}} = 30.7 - 0.23 (L/r) \text{ ksi} \quad L/r \leq 55$$

$$\sigma_{\text{allow}} = 54,000/(L/r)^2 \text{ ksi} \quad L/r \geq 55$$

<i>L</i> (ft)	6 ft	8 ft	10 ft	12 ft
<i>L/r</i>	39.05	52.06	65.08	78.09
σ_{allow} (ksi)	21.72	18.73	12.75	8.86
$P_{\text{allow}} = \sigma_{\text{allow}} A$	142 k	122 k	83 k	58 k

Problem 11.9-22 An aluminum pipe column (alloy 2014-T6) with pinned ends has outside diameter $d_2 = 120$ mm and inside diameter $d_1 = 110$ mm (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 1.0$ m, 2.0 m, 3.0 m, and 4.0 m.

(Hint: Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-22 Aluminum pipe column

Alloy 2014-T6

Pinned ends ($K = 1$).

$$d_2 = 120 \text{ mm} = 4.7244 \text{ in.}$$

$$d_1 = 110 \text{ mm} = 4.3307 \text{ in.}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 2.800 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 7.188 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 40.697 \text{ mm} = 1.6022 \text{ in.}$$

Use Eqs. (11-84 *a* and *b*):

$$\sigma_{\text{allow}} = 30.7 - 0.23 (L/r) \text{ ksi} \quad L/r \leq 55$$

$$\sigma_{\text{allow}} = 54,000/(L/r)^2 \text{ ksi} \quad L/r \geq 55$$

<i>L</i> (m)	1.0 m	2.0 m	3.0 m	4.0 m
<i>L</i> (in.)	39.37	78.74	118.1	157.5
<i>L/r</i>	24.58	49.15	73.73	98.30
σ_{allow} (ksi)	25.05	19.40	9.934	5.588
$P_{\text{allow}} = \sigma_{\text{allow}} A$	70.14 k	54.31 k	27.81 k	15.65 k
P_{allow} (kN)	312 kN	242 kN	124 kN	70 kN

Problem 11.9-23 An aluminum pipe column (alloy 6061-T6) that is fixed at the base and free at the top has outside diameter $d_2 = 3.25$ in. and inside diameter $d_1 = 3.00$ in. (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 2$ ft, 3 ft, 4 ft, and 5 ft.

Solution 11.9-23 Aluminum pipe column

Alloy 6061-T6

Fixed-free ends ($K = 2$).

$$d_2 = 3.25 \text{ in.}$$

$$d_1 = 3.00 \text{ in.}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.227 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.500 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 1.106 \text{ in.}$$

Use Eqs. (11-85 *a* and *b*):

$$\sigma_{\text{allow}} = 20.2 - 0.126 (KL/r) \text{ ksi} \quad KL/r \leq 66$$

$$\sigma_{\text{allow}} = 51,000/(KL/r)^2 \text{ ksi} \quad KL/r \geq 66$$

<i>L</i> (ft)	2 ft	3 ft	4 ft	5 ft
<i>KL/r</i>	43.40	65.10	86.80	108.5
σ_{allow} (ksi)	14.73	12.00	6.77	4.33
$P_{\text{allow}} = \sigma_{\text{allow}} A$	18.1 k	14.7 k	8.3 k	5.3 k

Problem 11.9-24 An aluminum pipe column (alloy 6061-T6) that is fixed at the base and free at the top has outside diameter $d_2 = 80$ mm and inside diameter $d_1 = 72$ mm (see figure).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 0.6$ m, 0.8 m, 1.0 m, and 1.2 m.

(Hint: Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-24 Aluminum pipe column

Alloy 6061-T6

Fixed-free ends ($K = 2$).

$$d_2 = 80 \text{ mm} = 3.1496 \text{ in.}$$

$$d_1 = 72 \text{ mm} = 2.8346 \text{ in.}$$

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = 1.480 \text{ in.}^2$$

$$I = \frac{\pi}{64}(d_2^4 - d_1^4) = 1.661 \text{ in.}^4$$

$$r = \sqrt{\frac{I}{A}} = 26.907 \text{ mm} = 1.059 \text{ in.}$$

Use Eqs. (11-85 *a* and *b*):

$$\sigma_{\text{allow}} = 20.2 - 0.126 (KL/r) \text{ ksi} \quad KL/r \leq 66$$

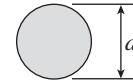
$$\sigma_{\text{allow}} = 51,000/(KL/r)^2 \text{ ksi} \quad KL/r \geq 66$$

L (m)	0.6 m	0.8 m	1.0 m	1.2 m
KL (in.)	47.24	62.99	78.74	94.49
KL/r	44.61	59.48	74.35	89.23
σ_{allow} (ksi)	14.58	12.71	9.226	6.405
$P_{\text{allow}} = \sigma_{\text{allow}} A$	21.58 k	18.81 k	13.65 k	9.48 k
P_{allow} (kN)	96 kN	84 kN	61 kN	42 kN

Problem 11.9-25 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 60$ k. The bar has pinned supports and is made of alloy 2014-T6.

(a) If the diameter $d = 2.0$ in., what is the maximum allowable length L_{max} of the bar?

(b) If the length $L = 30$ in., what is the minimum required diameter d_{min} ?



Probs. 11.9-25 through 11.9-28

Solution 11.9-25 Aluminum bar

Alloy 2014-T6

Pinned supports ($K = 1$). $P = 60$ k

(a) FIND L_{max} IF $d = 2.0$ IN.

$$A = \frac{\pi d^2}{4} = 3.142 \text{ in.}^2 \quad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.5 \text{ in.}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{60 \text{ k}}{3.142 \text{ in.}^2} = 19.10 \text{ ksi}$$

Assume L/r is less than 55:

$$\text{Eq. (11-84a): } \sigma_{\text{allow}} = 30.7 - 0.23 (L/r) \text{ ksi}$$

$$\text{or } 19.10 = 30.7 - 0.23 (L/r)$$

$$\text{Solve for } L/r: \frac{L}{r} = 50.43 \quad \frac{L}{r} < 55 \quad \therefore \text{ok}$$

$$L_{\text{max}} = (50.43)r = 25.2 \text{ in.} \quad \leftarrow$$

(b) FIND d_{min} IF $L = 30$ IN.

$$A = \frac{\pi d^2}{4} \quad r = \frac{d}{4} \quad \frac{L}{r} = \frac{30 \text{ in.}}{d/4} = \frac{120 \text{ in.}}{d}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{60 \text{ k}}{\pi d^2/4} = \frac{76.39}{d^2} \text{ (ksi)}$$

Assume L/r is greater than 55:

$$\text{Eq. (11-84b): } \sigma_{\text{allow}} = \frac{54,000 \text{ ksi}}{(L/r)^2}$$

$$\text{or } \frac{76.39}{d^2} = \frac{54,000}{(120/d)^2}$$

$$d^4 = 20.37 \text{ in.}^4 \quad d_{\text{min}} = 2.12 \text{ in.} \quad \leftarrow$$

$$L/r = 120/d = 120/2.12 = 56.6 > 55 \quad \therefore \text{ok}$$

Problem 11.9-26 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 175$ kN. The bar has pinned supports and is made of alloy 2014-T6.

(a) If the diameter $d = 40$ mm, what is the maximum allowable length L_{\max} of the bar?

(b) If the length $L = 0.6$ m, what is the minimum required diameter d_{\min} ?

(Hint: Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-26 Aluminum bar

Alloy 2014-T6

Pinned supports ($K = 1$). $P = 175$ kN = 39.34 k

(a) FIND L_{\max} IF $d = 40$ MM = 1.575 IN.

$$A = \frac{\pi d^2}{4} = 1.948 \text{ in.}^2 \quad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.3938 \text{ in.}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{39.34 \text{ k}}{1.948 \text{ in.}^2} = 20.20 \text{ ksi}$$

Assume L/r is less than 55:

$$\text{Eq. (11-84a): } \sigma_{\text{allow}} = 30.7 - 0.23 (L/r) \text{ ksi}$$

$$\text{or } 20.20 = 30.7 - 0.23 (L/r)$$

$$\text{Solve for } L/r: \quad \frac{L}{r} = 45.65 \quad \frac{L}{r} < 55 \quad \therefore \text{ok}$$

$$L_{\max} = (45.65) r = 17.98 \text{ in.} = 457 \text{ mm} \quad \leftarrow$$

(b) FIND d_{\min} IF $L = 0.6$ m = 23.62 in.

$$A = \frac{\pi d^2}{4} \quad r = \frac{d}{4} \quad \frac{L}{r} = \frac{23.62 \text{ in.}}{d/4} = \frac{94.48 \text{ in.}}{d}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{39.34 \text{ k}}{\pi d^2/4} = \frac{50.09}{d^2} \quad (\text{ksi})$$

Assume L/r is greater than 55:

$$\text{Eq. (11-84b): } \sigma_{\text{allow}} = \frac{54,000 \text{ ksi}}{(L/r)^2}$$

$$\text{or } \frac{50.09}{d^2} = \frac{54,000}{(94.48/d)^2}$$

$$d^4 = 8.280 \text{ in.}^4 \quad d_{\min} = 1.696 \text{ in.} = 43.1 \text{ mm} \quad \leftarrow$$

$$L/r = 94.48/d = 94.48/1.696 = 55.7 > 55 \quad \therefore \text{ok}$$

Problem 11.9-27 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 10$ k. The bar has pinned supports and is made of alloy 6061-T6.

(a) If the diameter $d = 1.0$ in., what is the maximum allowable length L_{\max} of the bar?

(b) If the length $L = 20$ in., what is the minimum required diameter d_{\min} ?

Solution 11.9-27 Aluminum bar

Alloy 6061-T6

Pinned Supports ($K = 1$). $P = 10$ k

(a) FIND L_{\max} IF $d = 1.0$ IN.

$$A = \frac{\pi d^2}{4} = 0.7854 \text{ in.}^2 \quad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2500 \text{ in.}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{10 \text{ k}}{0.7854 \text{ in.}^2} = 12.73 \text{ ksi}$$

Assume L/r is less than 66:

$$\text{Eq. (11-85a): } \sigma_{\text{allow}} = 20.2 - 0.126 (L/r) \text{ ksi}$$

$$\text{or } 12.73 = 20.2 - 0.126 (L/r)$$

$$\text{Solve For } L/r: \quad \frac{L}{r} = 59.29 \quad \frac{L}{r} < 66 \quad \therefore \text{ok}$$

$$L_{\max} = (59.29)r = 14.8 \text{ in.} \quad \leftarrow$$

(b) FIND d_{\min} IF $L = 20$ in.

$$A = \frac{\pi d^2}{4} \quad r = \frac{d}{4} \quad \frac{L}{r} = \frac{20 \text{ in.}}{d/4} = \frac{80 \text{ in.}}{d}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{10 \text{ k}}{\pi d^2/4} = \frac{12.73}{d^2} \text{ (ksi)}$$

Assume L/r is greater than 66:

$$\text{Eq. (11-85b): } \sigma_{\text{allow}} = \frac{51,000 \text{ ksi}}{(L/r)^2}$$

$$\text{or } \frac{12.73}{d^2} = \frac{51,000}{(80/d)^2}$$

$$d^4 = 1.597 \text{ in.}^4 \quad d_{\min} = 1.12 \text{ in.} \quad \leftarrow$$

$$L/r = 80/d = 80/1.12 = 71 > 66 \quad \therefore \text{ok}$$

Problem 11.9-28 A solid round bar of aluminum having diameter d (see figure) is compressed by an axial force $P = 60$ kN. The bar has pinned supports and is made of alloy 6061-T6.

(a) If the diameter $d = 30$ mm, what is the maximum allowable length L_{\max} of the bar?

(b) If the length $L = 0.6$ m, what is the minimum required diameter d_{\min} ?

(Hint: Convert the given data to USCS units, determine the required quantities, and then convert back to SI units.)

Solution 11.9-28 Aluminum bar

Alloy 6061-T6

Pinned Supports ($K = 1$). $P = 60 \text{ kN} = 13.49 \text{ k}$

(a) FIND L_{\max} IF $d = 30 \text{ mm} = 1.181 \text{ in.}$

$$A = \frac{\pi d^2}{4} = 1.095 \text{ in.}^2 \quad I = \frac{\pi d^4}{64}$$

$$r = \sqrt{\frac{I}{A}} = \frac{d}{4} = 0.2953 \text{ in.}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{13.49 \text{ k}}{1.095 \text{ in.}^2} = 12.32 \text{ ksi}$$

Assume L/r is less than 66:

$$\text{Eq. (11-85a): } \sigma_{\text{allow}} = 20.2 - 0.126 (L/r) \text{ ksi}$$

$$\text{or } 12.32 = 20.2 - 0.126 (L/r)$$

$$\text{Solve For } L/r: \quad \frac{L}{r} = 62.54 \quad \frac{L}{r} < 66 \quad \therefore \text{ok}$$

$$L_{\max} = (62.54)r = 18.47 \text{ in.} = 469 \text{ mm} \quad \leftarrow$$

(b) FIND d_{\min} IF $L = 0.6 \text{ m} = 23.62 \text{ in.}$

$$A = \frac{\pi d^2}{4} \quad r = \frac{d}{4} \quad \frac{L}{r} = \frac{23.62 \text{ in.}}{d/4} = \frac{94.48 \text{ in.}}{d}$$

$$\sigma_{\text{allow}} = \frac{P}{A} = \frac{13.48 \text{ k}}{\pi d^2/4} = \frac{17.18}{d^2} \text{ (ksi)}$$

Assume L/r is greater than 66:

$$\text{Eq. (11-85b): } \sigma_{\text{allow}} = \frac{51,000 \text{ ksi}}{(L/r)^2}$$

$$\text{or } \frac{17.18}{d^2} = \frac{51,000}{(94.48/d)^2}$$

$$d^4 = 3.007 \text{ in.}^4 \quad d_{\min} = 1.317 \text{ in.} = 33.4 \text{ mm} \quad \leftarrow$$

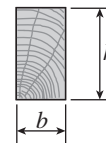
$$L/r = 94.48/d = 94.48/1.317 = 72 > 66 \quad \therefore \text{ok}$$

Wood Columns

When solving the problems for wood columns, assume that the columns are constructed of sawn lumber ($c = 0.8$ and $K_{cE} = 0.3$) and have pinned-end conditions. Also, buckling may occur about either principal axis of the cross section.

Problem 11.9-29 A wood post of rectangular cross section (see figure) is constructed of 4 in. \times 6 in. structural grade, Douglas fir lumber ($F_c = 2,000$ psi, $E = 1,800,000$ psi). The net cross-sectional dimensions of the post are $b = 3.5$ in. and $h = 5.5$ in. (see Appendix F).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 5.0$ ft, 7.5 ft, and 10.0 ft.



Probs. 11.9-29 through 11.9-32

Solution 11.9-29 Wood post (rectangular cross section)

$$F_c = 2,000 \text{ psi} \quad E = 1,800,000 \text{ psi} \quad c = 0.8$$

$$K_{cE} = 0.3 \quad b = 3.5 \text{ in.} \quad h = 5.5 \text{ in.} \quad d = b$$

Find P_{allow}

$$\text{Eq. (11-94): } \phi = \frac{K_{cE}E}{F_c(L_e/d)^2}$$

$$\text{Eq. (11-95): } C_p = \frac{1 + \phi}{2c} - \sqrt{\left[\frac{1 + \phi}{2c}\right]^2 - \frac{\phi}{c}}$$

$$\text{Eq. (11-92): } P_{\text{allow}} = F_c C_p A = F_c C_p b h$$

L_e	5 ft	7.5 ft	10.0 ft
L_e/d	17.14	25.71	34.29
ϕ	0.9188	0.4083	0.2297
C_p	0.6610	0.3661	0.2176
P_{allow}	25.4 k	14.1 k	8.4 k ←

Problem 11.9-30 A wood post of rectangular cross section (see figure) is constructed of structural grade, southern pine lumber ($F_c = 14 \text{ MPa}$, $E = 12 \text{ GPa}$). The cross-sectional dimensions of the post (actual dimensions) are $b = 100 \text{ mm}$ and $h = 150 \text{ mm}$.

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 1.5 \text{ m}$, 2.0 m , and 2.5 m .

Solution 11.9-30 Wood post (rectangular cross section)

$$F_c = 14 \text{ MPa} \quad E = 12 \text{ GPa} \quad c = 0.8 \quad K_{cE} = 0.3$$

$$b = 100 \text{ mm} \quad h = 150 \text{ mm} \quad d = b$$

Find P_{allow}

$$\text{Eq. (11-94): } \phi = \frac{K_{cE}E}{F_c(L_e/d)^2}$$

$$\text{Eq. (11-95): } C_p = \frac{1 + \phi}{2c} - \sqrt{\left[\frac{1 + \phi}{2c}\right]^2 - \frac{\phi}{c}}$$

$$\text{Eq. (11-92): } P_{\text{allow}} = F_c C_p A = F_c C_p b h$$

L_e	1.5 m	2.0 m	2.5 m
L_e/d	15	20	25
ϕ	1.1429	0.6429	0.4114
C_p	0.7350	0.5261	0.3684
P_{allow}	154 kN	110 kN	77 kN ←

Problem 11.9-31 A wood column of rectangular cross section (see figure) is constructed of 4 in. \times 8 in. construction grade, western hemlock lumber ($F_c = 1,000 \text{ psi}$, $E = 1,300,000 \text{ psi}$). The net cross-sectional dimensions of the column are $b = 3.5 \text{ in.}$ and $h = 7.25 \text{ in.}$ (see Appendix F).

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 6 \text{ ft}$, 8 ft , and 10 ft .

Solution 11.9-31 Wood column (rectangular cross section)

$$F_c = 1,000 \text{ psi} \quad E = 1,300,000 \text{ psi} \quad c = 0.8$$

$$K_{cE} = 0.3 \quad b = 3.5 \text{ in.} \quad h = 7.25 \text{ in.} \quad d = b$$

Find P_{allow}

$$\text{Eq. (11-94): } \phi = \frac{K_{cE}E}{F_c(L_e/d)^2}$$

$$\text{Eq. (11-95): } C_p = \frac{1 + \phi}{2c} - \sqrt{\left[\frac{1 + \phi}{2c}\right]^2 - \frac{\phi}{c}}$$

$$\text{Eq. (11-92): } P_{\text{allow}} = F_c C_p A = F_c C_p b h$$

L_e	6 ft	8 ft	10 ft
L_e/d	20.57	27.43	34.29
ϕ	0.9216	0.5184	0.3318
C_p	0.6621	0.4464	0.3050
P_{allow}	16.8 k	11.3 k	7.7 k ←

Problem 11.9-32 A wood column of rectangular cross section (see figure) is constructed of structural grade, Douglas fir lumber ($F_c = 12$ MPa, $E = 10$ GPa). The cross-sectional dimensions of the column (actual dimensions) are $b = 140$ mm and $h = 210$ mm.

Determine the allowable axial load P_{allow} for each of the following lengths: $L = 2.5$ m, 3.5 m, and 4.5 m.

Solution 11.9-32 Wood column (rectangular cross section)

$$F_c = 12 \text{ MPa} \quad E = 10 \text{ GPa} \quad c = 0.8 \quad K_{cE} = 0.3$$

$$b = 140 \text{ mm} \quad h = 210 \text{ mm} \quad d = b$$

Find P_{allow}

$$\text{Eq. (11-94): } \phi = \frac{K_{cE}E}{F_c(L_e/d)^2}$$

$$\text{Eq. (11-95): } C_p = \frac{1 + \phi}{2c} - \sqrt{\left[\frac{1 + \phi}{2c}\right]^2 - \frac{\phi}{c}}$$

$$\text{Eq. (11-92): } P_{\text{allow}} = F_c C_p A = F_c C_p b h$$

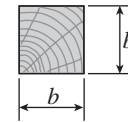
L_e	2.5 m	3.5 m	4.5 m
L_e/d	17.86	25.00	32.14
ϕ	0.7840	0.4000	0.2420
C_p	0.6019	0.3596	0.2284
P_{allow}	212 kN	127 kN	81 kN

←

Problem 11.9-33 A square wood column with side dimensions b (see figure) is constructed of a structural grade of Douglas fir for which $F_c = 1,700$ psi and $E = 1,400,000$ psi. An axial force $P = 40$ k acts on the column.

(a) If the dimension $b = 5.5$ in., what is the maximum allowable length L_{max} of the column?

(b) If the length $L = 11$ ft, what is the minimum required dimension b_{min} ?



Probs. 11.9-33 through 11.9-36

Solution 11.9-33 Wood column (square cross section)

$$F_c = 1,700 \text{ psi} \quad E = 1,400,000 \text{ psi} \quad c = 0.8$$

$$K_{cE} = 0.3 \quad P = 40 \text{ k}$$

(a) MAXIMUM LENGTH L_{max} FOR $b = d = 5.5$ IN.

$$\text{From Eq. (11-92): } C_p = \frac{P}{F_c b^2} = 0.77783$$

From Eq. (11-95):

$$C_p = 0.77783 = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6}\right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 1.3225$

$$\text{From Eq. (11-94): } \frac{L}{d} = \sqrt{\frac{K_{cE}E}{\phi F_c}} = 13.67$$

$$\therefore L_{\text{max}} = 13.67d = (13.67)(5.5 \text{ in.})$$

$$= 75.2 \text{ in.} \quad \leftarrow$$

(b) MINIMUM DIMENSION b_{min} FOR $L = 11$ ft

$$\text{Trial and error: } \frac{L}{d} = \frac{L}{b} \quad \phi = \frac{K_{cE}E}{F_c(L/d)^2}$$

$$C_p = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6}\right]^2 - \frac{\phi}{0.8}} \quad P = F_c C_p b^2$$

Given load: $P = 40$ k

Trial b (in.)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_p	P (kips)
6.50	20.308	0.59907	0.49942	35.87
6.70	19.701	0.63651	0.52230	39.86
6.71	19.672	0.63841	0.52343	40.06

$$\therefore b_{\text{min}} = 6.71 \text{ in.} \quad \leftarrow$$

Problem 11.9-34 A square wood column with side dimensions b (see figure) is constructed of a structural grade of southern pine for which $F_c = 10.5$ MPa and $E = 12$ GPa. An axial force $P = 200$ kN acts on the column.

(a) If the dimension $b = 150$ mm, what is the maximum allowable length L_{\max} of the column?

(b) If the length $L = 4.0$ m, what is the minimum required dimension b_{\min} ?

Solution 11.9-34 Wood column (square cross section)

$$F_c = 10.5 \text{ MPa} \quad E = 12 \text{ GPa} \quad c = 0.8$$

$$K_{cE} = 0.3 \quad P = 200 \text{ kN}$$

(a) MAXIMUM LENGTH L_{\max} FOR $b = d = 150$ mm

$$\text{From Eq. (11-92): } C_p = \frac{P}{F_c b^2} = 0.84656$$

From Eq. (11-95):

$$C_p = 0.84656 = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6} \right]^2 - \frac{\phi}{0.8}}$$

$$\text{Trial and error: } \phi = 1.7807$$

$$\text{From Eq. (11-94): } \frac{L}{d} = \sqrt{\frac{K_{cE} E}{\phi F_c}} = 13.876$$

$$\begin{aligned} \therefore L_{\max} &= 13.876 d = (13.876)(150 \text{ mm}) \\ &= 2.08 \text{ m} \quad \leftarrow \end{aligned}$$

(b) MINIMUM DIMENSION b_{\min} FOR $L = 4.0$ M

$$\text{Trial and error: } \frac{L}{d} = \frac{L}{b} \quad \phi = \frac{K_{cE} E}{F_c (L/d)^2}$$

$$C_p = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6} \right]^2 - \frac{\phi}{0.8}} \quad P = F_c C_p b^2$$

Given load: $P = 200$ kN

Trial b (mm)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_p	P (kN)
180	22.22	0.69429	0.55547	189.0
182	21.98	0.70980	0.56394	196.1
183	21.86	0.71762	0.56814	199.8
184	21.74	0.72549	0.57231	203.5

$$\therefore b_{\min} = 184 \text{ mm} \quad \leftarrow$$

Problem 11.9-35 A square wood column with side dimensions b (see figure) is constructed of a structural grade of spruce for which $F_c = 900$ psi and $E = 1,500,000$ psi. An axial force $P = 8.0$ k acts on the column.

(a) If the dimension $b = 3.5$ in., what is the maximum allowable length L_{\max} of the column?

(b) If the length $L = 10$ ft, what is the minimum required dimension b_{\min} ?

Solution 11.9-35 Wood column (square cross section)

$$F_c = 900 \text{ psi} \quad E = 1,500,000 \text{ psi} \quad c = 0.8$$

$$K_{cE} = 0.3 \quad P = 8.0 \text{ k}$$

(a) MAXIMUM LENGTH L_{\max} FOR $b = d = 3.5$ IN.

$$\text{From Eq. (11-92): } C_p = \frac{P}{F_c b^2} = 0.72562$$

From Eq. (11-95):

$$C_p = 0.72562 = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6} \right]^2 - \frac{\phi}{0.8}}$$

$$\text{Trial and error: } \phi = 1.1094$$

$$\text{From Eq. (11-94): } \frac{L}{d} = \sqrt{\frac{K_{cE} E}{\phi F_c}} = 21.23$$

$$\therefore L_{\max} = 21.23 d = (21.23)(3.5 \text{ in.}) = 74.3 \text{ in.} \quad \leftarrow$$

(b) MINIMUM DIMENSION b_{\min} FOR $L = 10$ FT

$$\text{Trial and error. } \frac{L}{d} = \frac{L}{b} \quad \phi = \frac{K_{cE}E}{F_c(L/d)^2}$$

$$C_P = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6} \right]^2 - \frac{\phi}{0.8}} \quad P = F_c C_P b^2$$

Given load: $P = 8000$ lb

Trial b (in.)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (lb)
4.00	30.00	0.55556	0.47145	6789
4.20	28.57	0.61250	0.50775	8061
4.19	28.64	0.60959	0.50596	7994

 $\therefore b_{\min} = 4.20$ in. ←

Problem 11.9-36 A square wood column with side dimensions b (see figure) is constructed of a structural grade of eastern white pine for which $F_c = 8.0$ MPa and $E = 8.5$ GPa. An axial force $P = 100$ kN acts on the column.

(a) If the dimension $b = 120$ mm, what is the maximum allowable length L_{\max} of the column?

(b) If the length $L = 4.0$ m, what is the minimum required dimension b_{\min} ?

Solution 11.9-36 Wood column (square cross section)

$$F_c = 8.0 \text{ MPa} \quad E = 8.5 \text{ GPa} \quad c = 0.8$$

$$K_{cE} = 0.3 \quad P = 100 \text{ kN}$$

(a) MAXIMUM LENGTH L_{\max} FOR $b = d = 120$ mm

$$\text{From Eq. (11-92): } C_P = \frac{P}{F_c b^2} = 0.86806$$

From Eq. (11-95):

$$C_P = 0.86806 = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6} \right]^2 - \frac{\phi}{0.8}}$$

Trial and error: $\phi = 2.0102$

$$\text{From Eq. (11-94): } \frac{L}{d} = \sqrt{\frac{K_{cE}E}{\phi F_c}} = 12.592$$

$$\therefore L_{\max} = 12.592 d = (12.592)(120 \text{ mm}) = 1.51 \text{ m} \quad \leftarrow$$

(b) MINIMUM DIMENSION b_{\min} FOR $L = 4.0$ m

$$\text{Trial and error. } \frac{L}{d} = \frac{L}{b} \quad \phi = \frac{K_{cE}E}{F_c(L/d)^2}$$

$$C_P = \frac{1 + \phi}{1.6} - \sqrt{\left[\frac{1 + \phi}{1.6} \right]^2 - \frac{\phi}{0.8}} \quad P = F_c C_P b^2$$

Given load: $P = 100$ kN

Trial b (mm)	$\frac{L}{d} = \frac{L}{b}$	ϕ	C_P	P (kN)
160	25.00	0.51000	0.44060	90.23
164	24.39	0.53582	0.45828	98.61
165	24.24	0.54237	0.46269	100.77

 $\therefore b_{\min} = 165$ mm ←

